

Pair creation of anti-de Sitter black holes on a cosmic string background

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We analyze the quantum process in which a cosmic string breaks in an anti-de Sitter (AdS) background, and a pair of charged or neutral black holes is produced at the ends of the strings. The energy to materialize and accelerate the pair comes from the strings tension. In an AdS background this is the only study done in the process of production of a pair of correlated black holes with spherical topology. The acceleration A of the produced black holes is necessarily greater than $\sqrt{|\Lambda|/3}$, where $\Lambda < 0$ is the cosmological constant. Only in this case the virtual pair of black holes can overcome the attractive background AdS potential well and become real. The instantons that describe this process are constructed through the analytical continuation of the AdS C-metric. Then, we explicitly compute the pair creation rate of the process, and we verify that (as occurs with pair creation in other backgrounds) the pair production of nonextreme black holes is enhanced relative to the pair creation of extreme black holes by a factor of $e^{\mathcal{A}_{bh}/4}$, where \mathcal{A}_{bh} is the black hole horizon area. We also conclude that the general behavior of the pair creation rate with the mass and acceleration of the black holes is similar in the AdS, flat and de Sitter cases, and our AdS results reduce to the ones of the flat case when $\Lambda \rightarrow 0$.

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I. INTRODUCTION

The quantum Schwinger-like process of black hole pair creation in an external field is by now a well studied subject. It supplied a way to produce Planck scale black holes, and it gave useful clues to the discussion of issues like the statistical properties of black holes, the black hole information paradox, and the analysis of topology changing processes.

In order to turn the pair of virtual black holes real one needs a background field that provides the energy needed to materialize the pair, and that furnishes the force necessary to accelerate away the black holes once they are created. This background field can be: (i) an external electromagnetic field with its Lorentz force (see [1]-[10]), (ii) the positive cosmological constant Λ , or inflation (see [11]-[17]), (iii) a cosmic string with its tension (see [18]-[22]), (iv) a domain wall with its gravitational repulsive energy (see [23]-[26]). One can also have a combination of the above fields, for example, a process involving cosmic string breaking in a background magnetic field [27], or a scenario in which a cosmic string breaks in a positive cosmological background [28].

The evaluation of the black hole pair creation rate has been done at the semiclassical level using the instanton method. An instanton is an Euclidean solution that interpolates between the initial and final states of a classically forbidden transition, and is a saddle point for the Euclidean path integral that describes the pair creation rate. The instantons that mediate the above processes can be obtained by analytically continuing (i) the Ernst

solution [29] (which is available only for $\Lambda = 0$), (ii) the de Sitter black hole solutions, (iii) the C-metric with a generalized cosmological constant Λ [30, 31], or (iv) the domain wall solution [32]. All these exact solutions describe appropriately the evolution of the black hole pair that has been created, since they represent a pair of black holes accelerated by an electromagnetic field, by a cosmological constant, by a string, or by a domain wall, respectively.

An important process that accompanies the production of the black hole pair is the emission of electromagnetic and gravitational radiation. In a flat background, an estimate for the amount of gravitational radiation released during the pair creation period has been given in [33]. After the pair creation, the black hole pair accelerates away and, consequently, the black holes continue to release gravitational and electromagnetic energy. In a $\Lambda = 0$ background, the gravitational radiation emitted by uniformly accelerated black holes has been computed in [34], while in a dS background this analysis has been carried in [35], and in an AdS background the programme has been completed in [36]. We ask the reader to see, e.g., the introduction of [28] for a recent and detailed description of the works done on black hole pair creation processes.

In this paper we want to analyze the process in which a cosmic string breaks and a pair of black holes is produced at the ends of the string, in an anti-de Sitter (AdS) background ($\Lambda < 0$). Therefore, the energy to materialize and accelerate the pair comes from the strings tension. In an AdS background this is the only study done in the process of production of a pair of correlated black holes with spherical topology. The analysis of this process in a flat background ($\Lambda = 0$) has been done in [18], while in a de Sitter background ($\Lambda > 0$) it has been carried in [28, 37].

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The instantons for this process can be constructed by analytically continuing the AdS C-metric found in [31] and analyzed in detail in [38, 39]. Contrary to the $\Lambda = 0$ [30] and $\Lambda > 0$ [37] cases, the AdS describes a pair of accelerated black holes only when the acceleration supplied by the strings is greater than $\sqrt{|\Lambda|/3}$ [38]. Hence we expect that pair creation of black holes in an AdS background is possible only when $A > \sqrt{|\Lambda|/3}$. We will confirm this expectation. The quantum production of uncorrelated AdS black holes has been studied in [40], and the pair creation process of topological AdS black holes (with hyperbolic topology) has been analyzed in [25] in a domain wall background.

The plan of this paper is as follows. In Sec. II we construct, from the AdS C-metric, the instantons that describe the pair creation process. In Sec. III, we explicitly evaluate the pair creation rate of black holes in an AdS background when a string breaks. In Sec. IV concluding remarks are presented. In the Appendix A, we discuss, for the benefit of comparison with the $\Lambda = 0$ case, the pair creation of black holes in a $\Lambda = 0$ background. We explicitly compute the numerical value of the pair creation rate, which has not been done yet. In the Appendix B a heuristic derivation of the pair creation rates is given. Throughout this paper we use units in which $G = c = \hbar = 1$.

II. THE AdS C-METRIC INSTANTONS

The AdS C-metric has been found in [31]. The physical properties and interpretation of this solution have been analyzed in [38, 39]. When $A > \sqrt{|\Lambda|/3}$, and only in this case [38], the AdS C-metric describes a pair of uniformly accelerated black holes in an AdS background, with the acceleration being provided by two strings, from each one of the black holes towards infinity, that pulls them away. Since we are interested in black hole pair creation, hereafter we will deal only with the $A > \sqrt{|\Lambda|/3}$ case. For a detailed discussion on the properties of the AdS C-metric we ask the reader to see [38]. Here we will only mention those which are really essential.

The gravitational field of the Lorentzian AdS C-metric is given by (see, e.g., [38])

$$ds^2 = [A(x+y)]^{-2} (-\mathcal{F}dt^2 + \mathcal{F}^{-1}dy^2 + \mathcal{G}^{-1}dx^2 + \mathcal{G}d\phi^2), \quad (1)$$

with

$$\begin{aligned} \mathcal{F}(y) &= -\frac{3A^2 - |\Lambda|}{3A^2} + y^2 - 2mAy^3 + q^2A^2y^4, \\ \mathcal{G}(x) &= 1 - x^2 - 2mAx^3 - q^2A^2x^4, \end{aligned} \quad (2)$$

where $\Lambda < 0$ is the cosmological constant, $A > 0$ is the acceleration of the black holes, and m and q are the ADM mass and electromagnetic charge of the non-accelerated black hole, respectively. The Maxwell field in the magnetic case is given by

$$F_{\text{mag}} = -q dx \wedge d\phi, \quad (3)$$

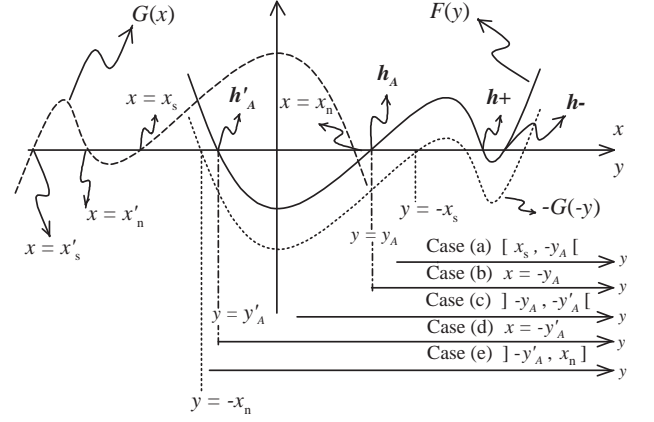


FIG. 1: Shape of $\mathcal{G}(x)$ and $\mathcal{F}(y)$ for a general nonextreme charged massive AdS C-metric. The allowed range of x is between x_s and x_n where $\mathcal{G}(x)$ is positive and compact. In the Lorentzian sector, the permitted range of y is $-x \leq y < +\infty$. The presence of an accelerated horizon is indicated by y_A and y'_A and the inner and outer charged horizons by y_- and y_+ . In the extreme case y_- and y_+ coincide, and $\mathcal{G}(x)$ has only two real roots represented by x_s and x_n . When $m = 0$ and $q = 0$, both $\mathcal{G}(x)$ and $\mathcal{F}(y)$ are represented by a parabola with the only zeros of $\mathcal{G}(x)$ being x_s and x_n , and the only zeros of $\mathcal{F}(x)$ being y'_A and y_A .

while in the electric case it is given by

$$F_{\text{el}} = -q dt \wedge dy. \quad (4)$$

The general shape of $\mathcal{G}(x)$ and $\mathcal{F}(y)$ is represented in Fig. 1.

The solution has a curvature singularity at $y = +\infty$ where the matter source is. The point $y = -x$ corresponds to a point that is infinitely far away from the curvature singularity, thus as y increases we approach the curvature singularity and $y + x$ is the inverse of a radial coordinate. At most, $\mathcal{F}(y)$ can have four real zeros which we label in ascending order by $y_{\text{neg}} < 0 < y_A \leq y_+ \leq y_-$. The roots y_- and y_+ are respectively the inner and outer charged black hole horizons, and y_A is an acceleration horizon. The negative root y_{neg} satisfies $y_{\text{neg}} < -x$ and thus has no physical significance. The angular coordinate x belongs to the range $[x_s, x_n]$ for which $\mathcal{G}(x) \geq 0$ (when we set $A = 0$ we have $x_s = -1$ and $x_n = +1$). In order to avoid a conical singularity in the north pole, the period of ϕ must be given by

$$\Delta\phi = \frac{4\pi}{|\mathcal{G}'(x_n)|}, \quad (5)$$

and this leaves a conical singularity in the south pole with deficit angle

$$\delta = 2\pi \left(1 - \frac{\mathcal{G}'(x_s)}{|\mathcal{G}'(x_n)|} \right). \quad (6)$$

that signals the presence of strings from each one of the black holes towards infinity with mass density $\mu =$

$\delta/(8\pi)$, and with pressure $p = -\mu < 0$. When we set the acceleration parameter A equal to zero, the AdS C-metric reduces to the usual AdS–Reissner–Nordström or AdS–Schwarzschild solutions without conical singularities.

Later on it will be crucial to note that the number and nature of the horizons crossed by an observer that travels into the black hole singularity depends on the angular direction x that he is following (see Fig. 1). This peculiar feature is due to the lower restriction on the value of y ($-x \leq y$), and a detailed discussion and explanation of it can be found in [38] (such an angular dependence occurs also in the flat C-metric [30], although not so sharply). For example, when the observer is travelling towards the black hole singularity following an angular direction in the vicinity of the south pole ($x_s \leq x < -y_A$), he will cross only the outer y_+ and inner y_- black hole horizons (see Fig. 1), while when he does this trip following an angular direction in the vicinity of the equator ($-y_A < x < -y'_A$), he crosses the acceleration horizon y_A before passing through the black hole horizons y_+ and y_- .

In order to evaluate the black hole pair creation rate we need to find the instantons of the theory, i.e., we must look into the Euclidean section of the AdS C-metric and choose those Euclidean solutions which are regular in a way that will be explained soon. The Euclidean section of the AdS C-metric is obtained from the Lorentzian AdS C-metric by introducing an imaginary time coordinate $\tau = -it$ in (1)–(4). To have a positive definite Euclidean metric we must require that y belongs to $y_A \leq y \leq y_+$. In general, when $y_+ \neq y_-$, one then has conical singularities at the horizons $y = y_A$ and $y = y_+$. In order to obtain a regular solution we have to eliminate the conical singularities at both horizons, ensuring in this way that the system is in thermal equilibrium. This is achieved by imposing that the period of τ is the same for the two horizons, and is equivalent to requiring that the Hawking temperature of the two horizons be equal. To eliminate the conical singularity at $y = y_A$ the period of τ must be $\beta = 2\pi/k_A$ (where k_A is the surface gravity of the acceleration horizon),

$$\beta = \frac{4\pi}{|\mathcal{F}'(y_A)|}. \quad (7)$$

This choice for the period of τ also eliminates simultaneously the conical singularity at the outer black hole horizon, y_+ , if and only if the parameters of the solution are such that the surface gravities of the black hole and acceleration horizons are equal ($k_+ = k_A$), i.e.

$$\mathcal{F}'(y_+) = -\mathcal{F}'(y_A). \quad (8)$$

This condition is satisfied by a regular Euclidean solution with $y_A \neq y_+$ that will be referred to as nonextreme AdS instanton.

We now turn our attention to the case $y_+ = y_-$ (and $y_A \neq y_+$). When this happens the allowed range of y in the Euclidean sector is simply $y_A \leq y < y_+$. This occurs because when $y_+ = y_-$ the proper distance along spatial

directions between y_A and y_+ goes to infinity. The point y_+ disappears from the τ, y section which is no longer compact but becomes topologically $S^1 \times \mathbb{R}$. Thus, in this case we have a conical singularity only at y_A , and so we obtain a regular Euclidean solution by simply requiring that the period of τ be equal to (7). We will label this solution by extreme AdS instanton.

In a de Sitter (dS) background there is another $y_+ \neq y_-$ instanton which satisfies $y_A = y_+$. It is called Nariai instanton. Moreover, in the dS background, there is also a special solution that satisfies $y_A = y_+ = y_-$. It is called ultracold instanton. In the AdS C-metric case, the counterparts of these dS instantons are of no interest for the pair creation process because they are out of the allowed range of the angular direction x .

In Secs. IIA and IIB we will describe in detail the nonextreme AdS instanton with $m = q$ and the extreme AdS instanton with $y_+ = y_-$, respectively. These instantons are the natural AdS C-metric counterparts of the lukewarm dS C and cold dS C instantons constructed in [37]. Thus, these instantons could also be labelled as lukewarm AdS C and cold AdS C instantons. Moreover, in Sec. IIC, we will also describe saddle point solutions that have conical singularities. These solutions represent nonextreme instantons with $m \neq q$. In Sec. IID we will study the background reference spacetime that describes the initial system before the pair creation. These results will allow us to calculate the pair creation rate of accelerated AdS black holes in Sec. III.

A. The nonextreme AdS instanton with $m = q$

As we said above, for the nonextreme AdS instanton the gravitational field is given by (1) with the requirement that $\mathcal{F}(y)$ satisfies $\mathcal{F}(y_+) = 0 = \mathcal{F}(y_A)$ and $\mathcal{F}'(y_+) = -\mathcal{F}'(y_A)$. In this case we can then write

$$\begin{aligned} \mathcal{F}(y) = & -\left(\frac{y_A y_+}{y_A + y_+}\right)^2 \left(1 - \frac{y}{y_A}\right) \left(1 - \frac{y}{y_+}\right) \\ & \times \left(1 + \frac{y_A + y_+}{y_A y_+} y - \frac{y^2}{y_A y_+}\right), \end{aligned} \quad (9)$$

with

$$\begin{aligned} y_A = \frac{1 - \alpha}{2mA}, \quad y_+ = \frac{1 + \alpha}{2mA}, \\ \text{and } \alpha = \sqrt{1 - \frac{4m}{\sqrt{3}} \sqrt{3A^2 - |\Lambda|}}. \end{aligned} \quad (10)$$

The parameters A , Λ , m and q , written as a function of y_A and y_+ , are

$$\begin{aligned} \frac{|\Lambda|}{3A^2} &= \left(\frac{y_A y_+}{y_A + y_+}\right)^2, \\ mA &= (y_A + y_+)^{-1} = qA. \end{aligned} \quad (11)$$

Thus, the mass and the charge of the nonextreme AdS instanton are necessarily equal, $m = q$, as occurs with

its flat [4, 5, 9] and dS counterparts [11, 12, 37]. The demand that α is real requires that

$$0 < m \leq \frac{1}{4} \sqrt{\frac{3}{3A^2 - |\Lambda|}}, \quad (12)$$

and that

$$A > \sqrt{|\Lambda|/3}. \quad (13)$$

Therefore, as already anticipated, the nonextreme AdS instanton is available only when (13) is satisfied.

As we said, the allowed range of y in the Euclidean sector is $y_A \leq y \leq y_+$. Then, the period of τ , (7), that avoids the conical singularity at both horizons is

$$\beta = \frac{8\pi mA}{\alpha(1 - \alpha^2)}, \quad (14)$$

and $T = 1/\beta$ is the common temperature of the two horizons.

Using the fact that $\mathcal{G}(x) = |\Lambda|/(3A^2) - \mathcal{F}(-x)$ [see (2)] we can write

$$\mathcal{G}(x) = 1 - x^2(1 + mA x)^2, \quad (15)$$

and the roots of $\mathcal{G}(x)$ we are interested in are the south and north pole (represented, respectively, as x_s and x_n in Fig. 1),

$$x_s = \frac{-1 + \omega_-}{2mA} < 0, \quad x_n = \frac{-1 + \omega_+}{2mA} > 0, \\ \text{with } \omega_{\pm} = \sqrt{1 \pm 4mA}. \quad (16)$$

When m and q go to zero we have $x_s \rightarrow -1$ and $x_n \rightarrow +1$. This is the reason why we decided to work in between the roots x_s and x_n , instead of working in between the roots x'_s and x'_n also represented in Fig. 1. Indeed, when $m \rightarrow 0$ and $q \rightarrow 0$ these two last roots disappear, and our instanton has no vacuum counterpart. Now, the requirement that ω_- is real demands that $mA < 1/4$ [note that $1/(4A) < (1/4)\sqrt{3/(3A^2 - |\Lambda|)}$, see (12)]. If this requirement is not fulfilled then $\mathcal{G}(x)$ has only two real roots that are represented as x'_s and x_n in Fig. 1 and, as we have just said, in this case the solution has no counterpart in the $m = 0$ and $q = 0$ case. Therefore we discard the solutions that satisfy $\frac{1}{4A} \leq m \leq \frac{1}{4}\sqrt{\frac{3}{3A^2 - |\Lambda|}}$, and hereafter when we refer to the mass of the nonextreme AdS instanton we will be working in the range

$$0 < m \leq \frac{1}{4A}, \quad (17)$$

The period of ϕ , (5), that avoids the conical singularity at the north pole (and leaves one at the south pole responsible for the presence of the string) is

$$\Delta\phi = \frac{8\pi mA}{\omega_+(\omega_+^2 - 1)} < 2\pi. \quad (18)$$

When m and q go to zero we have $\Delta\phi \rightarrow 2\pi$ and the conical singularity disappears.

The topology of the nonextreme AdS instanton is $S^2 \times S^2 - \{\text{region}\}$, where $S^2 \times S^2$ represents $0 \leq \tau \leq \beta$, $y_A \leq y \leq y_+$, $x_s \leq x \leq x_n$, and $0 \leq \phi \leq \Delta\phi$, but we have to remove the region, $\{\text{region}\} = \{x, y : x_s \leq x \leq -y_A \wedge y + x = 0\}$. The Lorentzian sector of this nonextreme instanton describes two charged AdS black holes being accelerated by the strings, so this instanton describes pair creation of nonextreme black holes with $m = q$.

B. The extreme AdS instanton with $y_+ = y_-$

The gravitational field of the extreme AdS instanton is given by (1) with the requirement that the size of the outer charged black hole horizon y_+ is equal to the size of the inner charged horizon y_- . Let us label this degenerated horizon by ρ : $y_+ = y_- \equiv \rho$ and $\rho > y_A$. In this case, the function $\mathcal{F}(y)$ can be written as

$$\mathcal{F}(y) = \frac{\rho^2 - 3\gamma}{\rho^4} (y - y'_A)(y - y_A)(y - \rho)^2, \quad (19)$$

with

$$\gamma = \frac{3A^2 - |\Lambda|}{3A^2}, \quad \text{and } A > \sqrt{|\Lambda|/3}. \quad (20)$$

Note that, as occurred with the nonextreme AdS instanton, the extreme AdS instanton is also available only when $A > \sqrt{|\Lambda|/3}$. The roots ρ , y'_A and y_A are given by

$$\rho = \frac{3m}{4q^2 A} \left(1 + \sqrt{1 - \frac{8}{9} \frac{q^2}{m^2}} \right), \quad (21)$$

$$y'_A = \frac{\gamma\rho}{\rho^2 - 3\gamma} \left(1 - \sqrt{\frac{\rho^2 - 2\gamma}{\gamma}} \right), \quad (22)$$

$$y_A = \frac{\gamma\rho}{\rho^2 - 3\gamma} \left(1 + \sqrt{\frac{\rho^2 - 2\gamma}{\gamma}} \right). \quad (23)$$

The mass and the charge of the solution are written as a function of ρ as

$$m = \frac{1}{A\rho} \left(1 - \frac{2\gamma}{\rho^2} \right), \\ q^2 = \frac{1}{A^2\rho^2} \left(1 - \frac{3\gamma}{\rho^2} \right), \quad (24)$$

and, for a fixed A and Λ , the ratio q/m is higher than 1. The conditions $\rho > y_A$ and $q^2 > 0$ require that, for the extreme AdS instanton, the allowed range of ρ is

$$\rho > \sqrt{6\gamma}. \quad (25)$$

The value of y_A decreases monotonically with ρ and we have $\sqrt{\gamma} < y_A < \sqrt{6\gamma}$. The mass and the charge of the

extreme AdS instanton are also monotonically decreasing functions of ρ , and as we come from $\rho = +\infty$ into $\rho = \sqrt{6}\gamma$ we have

$$0 < m < \frac{\sqrt{2}}{3} \frac{1}{\sqrt{3A^2 - |\Lambda|}}, \quad (26)$$

$$0 < q < \frac{1}{2} \frac{1}{\sqrt{3A^2 - |\Lambda|}}, \quad (27)$$

so, for a fixed Λ , as the acceleration parameter A grows the maximum value of the mass and of the charge decreases monotonically. For a fixed Λ and for a fixed mass below $\sqrt{2/(9|\Lambda|)}$, the maximum value of the acceleration is $\sqrt{2/(27m^2) + |\Lambda|/3}$.

As we have already said, the allowed range of y in the Euclidean sector is $y_A \leq y < y_+$ and does not include $y = y_+$. Then, the period of τ , (7), that avoids the conical singularity at the only horizon of the extreme AdS instanton is

$$\beta = \frac{2\pi\rho^3}{(y_A - \rho)^2 \sqrt{\gamma(\rho^2 - 2\gamma)}}, \quad (28)$$

and $T = 1/\beta$ is the temperature of the acceleration horizon.

In what concerns the angular sector of the extreme AdS instanton, $\mathcal{G}(x)$ is given by (2), and its only real zeros are the south and north pole (represented, respectively, as x_s and x_n in Fig. 1; x'_s and x'_n also represented in this figure become complex roots in the extreme case),

$$\begin{aligned} x_s &= -p + \frac{h}{2} - \frac{m}{2q^2A} < 0, \\ x_n &= +p + \frac{h}{2} - \frac{m}{2q^2A} > 0, \end{aligned} \quad (29)$$

with

$$\begin{aligned} p &= \frac{1}{2} \left(-\frac{s}{3} + \frac{2m^2}{q^4A^2} - \frac{1 - 12q^2A^2}{3sq^4A^4} - \frac{4}{3q^2A^2} + n \right)^{1/2}, \\ n &= \frac{-m^3 + mq^2}{2hq^6A^3}, \\ h &= \sqrt{\frac{s}{3} + \frac{m^2}{q^4A^2} + \frac{1 - 12q^2A^2}{3sq^4A^4} - \frac{2}{3q^2A^2}}, \\ s &= \frac{1}{2^{1/3}q^2A^2} \left(\lambda - \sqrt{\lambda^2 - 4(1 - 12q^2A^2)^3} \right)^{1/3}, \\ \lambda &= 2 - 108m^2A^2 + 72q^2A^2, \end{aligned} \quad (30)$$

where m and q are fixed by (24), for a given A , Λ and ρ . When $m \rightarrow 0$ and $q \rightarrow 0$ we have $x_s \rightarrow -1$ and $x_n \rightarrow +1$. The period of ϕ that avoids the conical singularity at the north pole (and leaves one at the south pole responsible for the presence of the strings) is given by (5) with x_n defined in (29).

The topology of the extreme AdS instanton is $\mathbb{R}^2 \times S^2 - \{region\}$, where $\mathbb{R}^2 \times S^2$ represents $0 \leq \tau \leq \beta$,

$y_A \leq y < y_+$, $x_s \leq x \leq x_n$, and $0 \leq \phi \leq \Delta\phi$, but we have to remove the region, $\{region\} = \{(x, y) : x_s \leq x \leq -y_A \wedge y + x = 0\}$. Since $y = y_+ = \rho$ is at an infinite proper distance, the surface $y = y_+ = \rho$ is an internal infinity boundary. The Lorentzian sector of this extreme case describes two extreme ($y_+ = y_-$) charged AdS black holes being accelerated by the strings, and the extreme AdS instanton describes the pair creation of these extreme black holes.

C. The nonextreme AdS instanton with $m \neq q$

The AdS C-metric instantons studied in the two last subsections, namely the nonextreme instantons with $m = q$ and the extreme instantons with $y_+ = y_-$, are saddle point solutions free of conical singularities both in the y_+ and y_A horizons. The corresponding black holes may then nucleate in the AdS background when a cosmic string breaks, and we will compute their pair creation rates in Sec. III. However, these particular black holes are not the only ones that can be pair created. Indeed, it has been shown in [41] that Euclidean solutions with conical singularities may also be used as saddle points for the pair creation process. These nonextreme instantons have $m \neq q$ and describe pair creation of nonextreme black holes with $m \neq q$.

In what follows we will find the range of parameters for which one has nonextreme black holes with conical singularities, i.e., with $m \neq q$. First, when $m \neq 0$ and $q \neq 0$, we require that x belongs to the interval $[x_s, x_n]$ (sketched in Fig. 1) for which the charged solutions are in the same sector of the $m = 0$ and $q = 0$ solutions. Defining

$$\begin{aligned} \chi &\equiv \frac{q^2}{m^2}, \quad 0 < \chi \leq \frac{9}{8}, \quad \gamma_{\pm} \equiv 1 \pm \sqrt{1 - \frac{8}{9}\chi}, \\ \sigma(\chi, \gamma_{\pm}) &= \frac{(4\chi)^2(3\gamma_{\pm})^2 - 8\chi(3\gamma_{\pm})^3 + \chi(3\gamma_{\pm})^4}{(4\chi)^4}, \end{aligned} \quad (31)$$

the above requirement is fulfilled by the parameter range

$$m^2A^2 < \sigma(\chi, \gamma_-), \quad (32)$$

for which $\mathcal{G}(x = x_0) < 0$, with $x_0 = -\frac{3\gamma_-}{4\chi} \frac{1}{mA}$ being the less negative x where the derivative of $\mathcal{G}(x)$ vanishes. Second, in order to insure that one has a nonextreme solution we must require that

$$m^2A^2 > \sigma(\chi, \gamma_+) + m^2|\Lambda|/3, \quad (33)$$

for which $\mathcal{F}(y = y_0) < 0$, with $y_0 = \frac{3\gamma_+}{4\chi} \frac{1}{mA}$ being the point in between y_+ and y_- where the derivative of $\mathcal{F}(y)$ vanishes. We have $\sigma(\chi, \gamma_-) > \sigma(\chi, \gamma_+)$ except at $\chi = 9/8$ where these two functions are equal; $\sigma(\chi, \gamma_-)$ is always positive; and $\sigma(\chi, \gamma_+) < 0$ for $0 < \chi < 1$ and $\sigma(\chi, \gamma_+) > 0$ for $1 < \chi \leq 9/8$. The nonextreme black

holes with conical singularities are those that satisfy (32), (33) and $A > \sqrt{|\Lambda|/3}$. The range of parameters of these nonextreme black holes with $m \neq q$ are sketched in Fig. 2.

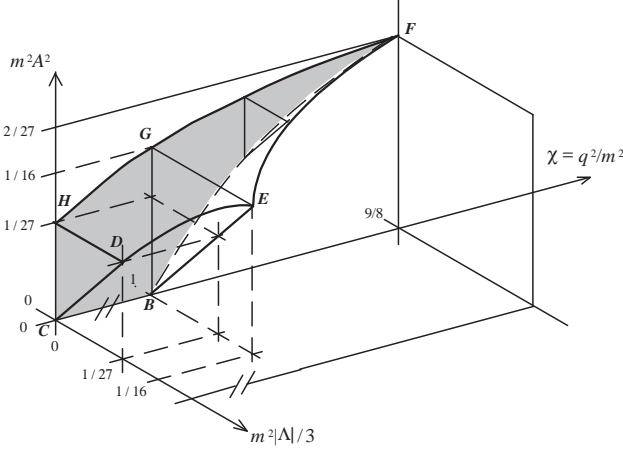


FIG. 2: Allowed ranges of the parameters $\Lambda, A, m, \chi \equiv q^2/m^2$ for which one has a solution representing a pair of accelerated black holes. The planar surface whose frontier is the triangle BEG represents the nonextreme AdS instanton with $m = q$. The curved surface delimited by the closed line BEF represents the extreme AdS instanton with $y_+ = y_-$ and satisfies $m^2 A^2 = \sigma(\chi, \gamma_+) + m^2 |\Lambda|/3$. The curved surface whose frontier is $DEFGH$ satisfies $m^2 A^2 = \sigma(\chi, \gamma_-)$ [see (32)]. The plane surface with boundary given by $BCDE$ satisfies $A = \sqrt{|\Lambda|/3}$. Neutral AdS instantons ($q = 0$) are those that belong to the planar surface with the triangle boundary CDH . The nonextreme AdS instantons with $m \neq q$ are those whose parameters are in the volume with boundary defined by $BCDE, BEF, CBFGH, CDH$ and $DEFGH$.

In order to compute the pair creation rate of the nonextreme black holes with $m \neq q$, we will need the relation between the parameters A, Λ, m, q , and the horizons y_A, y_+ and y_- . In general, for a nonextreme solution with horizons $y_A < y_+ < y_-$, one has

$$\mathcal{F}(y) = -\frac{1}{d}(y - y_A)(y - y_+)(y - y_-)(ay + b), \quad (34)$$

with

$$\begin{aligned} d &= y_A y_+ y_- (y_A + y_+ + y_-) + (y_A y_+ + y_A y_- + y_+ y_-)^2 \\ a &= (y_A y_+ + y_A y_- + y_+ y_-) \\ b &= y_A y_+ y_- . \end{aligned} \quad (35)$$

The parameters A, Λ, m and q can be expressed as a function of y_A, y_+ and y_- by

$$\begin{aligned} \frac{|\Lambda|}{3A^2} &= 1 - d^{-2}(y_A y_+ y_-)^2 \\ q^2 A^2 &= d^{-1}(y_A y_+ + y_A y_- + y_+ y_-) \\ mA &= (2\sigma)^{-1}(y_A + y_+)(y_A + y_-)(y_+ + y_-) \\ \sigma &= y_A^2 y_+ y_- + y_A y_+^2 y_- + y_A y_+ y_-^2 + (y_A y_+)^2 \\ &\quad + (y_A y_-)^2 + (y_+ y_-)^2 . \end{aligned} \quad (36)$$

The allowed values of parameters m and q are those contained in the interior region sketched in Fig. 2.

The topology of these nonextreme AdS instantons with $m \neq q$ is $S^2 \times S^2 - \{\text{region}\}$, where $S^2 \times S^2$ represents $0 \leq \tau \leq \beta, y_A \leq y \leq y_+, x_s \leq x \leq x_n$, and $0 \leq \phi \leq \Delta\phi$, but we have to remove the region, $\{\text{region}\} = \{\{x, y\} : x_s \leq x \leq -y_A \wedge y + x = 0\}$. These instantons describe the pair creation of nonextreme black holes with $m \neq q$.

D. Initial system: AdS background with a string

So far, we have described the solution that represents a pair of black holes accelerated by the strings tension. This solution describes the evolution of the black hole pair after its creation. Now, we want to find a solution that represents a string in a AdS background. This solution will describe the initial system, before the breaking of the cosmic string that leads to the formation of the black hole pair. In order to achieve our aim we note that at spatial infinity the gravitational field of the Euclidean AdS C-metric reduces to

$$ds^2 = \frac{1}{[A_0(x+y)]^2} \left[- \left(\frac{|\Lambda|}{3A_0^2} - 1 + y^2 \right) dt^2 + \frac{dy^2}{\frac{|\Lambda|}{3A_0^2} - 1 + y^2} + \frac{dx^2}{1 - x^2} + (1 - x^2) d\phi_0^2 \right], \quad (37)$$

and the Maxwell field goes to zero. A_0 is a constant that represents a freedom in the choice of coordinates, and $-1 \leq x \leq 1$. We want that this metric also describes the solution before the creation of the black hole pair, i.e., we demand that it describes a string with its conical deficit in an AdS background. Now, if we want to maintain the intrinsic properties of the string during the process we must impose that its mass density and thus its conical deficit remains constant. After the pair creation we already know that the conical deficit is given by (6). Hence, the requirement that the background solution describes an AdS spacetime with a conical deficit angle given exactly by (6) leads us to impose that in (37) one has

$$\Delta\phi_0 = 2\pi - \delta = 2\pi \frac{\mathcal{G}'(x_s)}{|\mathcal{G}'(x_n)|}. \quad (38)$$

The arbitrary parameter A_0 can be fixed as a function of A by imposing a matching between (1) and (37) at large spatial distances. We will do this matching in the following section.

III. CALCULATION OF THE BLACK HOLE PAIR CREATION RATES

The black hole pair creation rate is given by the path integral

$$\Gamma(g_{ij}, A_i) = \int d[g_{\mu\nu}] d[A_\mu] e^{-[I(g_{\mu\nu}, A_\mu) - I_0(g_{\mu\nu}^0, A_\mu^0)]}, \quad (39)$$

where g_{ij} and A_i are the induced metric and electromagnetic potential on the boundary $\partial\mathcal{M}$ of a compact manifold \mathcal{M} , $d[g_{\mu\nu}]$ is a measure on the space of the metrics $g_{\mu\nu}$ and $d[A_\mu]$ is a measure on the space of Maxwell field A_μ , and $I(g_{\mu\nu}, A_\mu)$ is the Euclidean action of the instanton that mediates the process. In our case this action is the Einstein-Maxwell action with a negative cosmological constant Λ . The path integral is over all compact metrics and potentials on manifolds \mathcal{M} with boundary $\partial\mathcal{M}$, which agree with the boundary data on $\partial\mathcal{M}$. $I_0(g_{\mu\nu}^0, A_\mu^0)$ is the action for the background reference spacetime, the AdS background with the string, specified by $g_{\mu\nu}^0$ and A_μ^0 (see Sec. IID). Its presence is required because it describes the initial system. Moreover, the geometry of the final system with the black hole pair is noncompact, that is $I(g_{\mu\nu}, A_\mu)$ diverges. However, the physical action $I(g_{\mu\nu}, A_\mu) - I_0(g_{\mu\nu}^0, A_\mu^0)$ is finite for fields $g_{\mu\nu}, A_\mu$ that approach asymptotically $g_{\mu\nu}^0, A_\mu^0$ in an appropriate way [42]. Specifically, one fixes a boundary near infinity, Σ^∞ , and demands that $g_{\mu\nu}, A_\mu$ and $g_{\mu\nu}^0, A_\mu^0$ induce fields on this boundary that agree to sufficient order, so that their difference does not contribute to the action in the limit that Σ^∞ goes to infinity [42].

In the semiclassical instanton approximation, the dominant contribution to the path integral (39) comes from metrics and Maxwell fields which are near the solutions (instantons) that extremalize the Euclidean action and satisfy the boundary conditions. In this approximation, the pair creation rate of AdS black holes is then given by

$$\Gamma \sim \eta e^{-I_{\text{inst}}}, \quad (40)$$

where $I_{\text{inst}} \equiv I - I_0$ includes already the contribution from the background reference spacetime. η is the one-loop prefactor that accounts for the fluctuations in the gravitational and matter fields, and its evaluation will not be considered in this paper (see [14, 15, 43, 44] for a treatment of this factor in some backgrounds).

Hawking and Horowitz [42] (see also Brown and York [45]) have shown that the Euclidean action of the instanton that mediates pair creation of nonextreme black holes can be written in the form

$$I_{\text{inst}} = \beta H - \frac{1}{4} (\Delta\mathcal{A}_{\text{ac}} + \mathcal{A}_{\text{bh}}), \quad (41)$$

where $\Delta\mathcal{A}_{\text{ac}}$ is the difference in area of the acceleration horizon between the AdS C-metric and the background, and \mathcal{A}_{bh} is the area of the black hole horizon present in the instanton. The Euclidean action of the instanton that describes pair creation of extreme black holes can be written as

$$I_{\text{inst}} = \beta H - \frac{1}{4} \Delta\mathcal{A}_{\text{ac}}. \quad (42)$$

In these relations, β is the period of the Euclidean time, and H represents the Hamiltonian of the system which, for static solutions, is given by [42, 45]

$$H = \int_{\Sigma_t} d^3x \sqrt{h} N \mathcal{H} - \frac{1}{8\pi} \int_{\Sigma_t^\infty} d^2x \sqrt{\sigma} (N^2 K - N_0^2 K_0).$$

The boundary ∂M consists of an initial and final space-like surface, Σ_t , of constant t with unit normal u^μ ($u \cdot u = -1$) and with intrinsic metric $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ plus a timelike surface near infinity, Σ^∞ , with unit normal n^μ ($n \cdot n = 1$, and $u \cdot n = 0$) and with intrinsic metric $\sigma_{\mu\nu} = h_{\mu\nu} - n_\mu n_\nu$. This surface Σ^∞ needs not to be at infinity. It can also be at a black hole horizon or at an internal infinity, but we will generally label it by Σ^∞ . This surface Σ^∞ is foliated by a family of 2-surfaces Σ_t^∞ that result from the intersection between Σ^∞ and Σ_t . In (43), N and N_0 are the lapse functions of the system with the pair and of the background, respectively, and $N = N_0$ in the boundary near infinity, Σ^∞ . \mathcal{H} is the hamiltonian constraint that contains contributions from the gravitational and Maxwell fields, and vanishes for solutions of the equations of motion. Then, the Hamiltonian is simply given by the boundary surface term. In this surface term, 2K represents the trace of the extrinsic curvature of the surface imbedded in the AdS C-metric, and 2K_0 is the extrinsic curvature of the surface imbedded in the background spacetime.

We will now verify that the boundary surface term in the Hamiltonian (43) is also zero, and thus the Hamiltonian makes no contribution to (41) and (42). We follow the technical procedure applied by Hawking, Horowitz and Ross [8] in the Ernst solution and by Hawking and Ross [18] in the flat C-metric. As we said above, one will require that the intrinsic metric $ds_{(\Sigma_t^\infty)}^2$ on the boundary Σ_t^∞ as embedded in the AdS C-metric agrees (to sufficient order) with the intrinsic metric on the boundary Σ_t^∞ as embedded in the background spacetime, in order to be sure that one is taking the same near infinity boundary in the evaluation of the quantities in the two spacetimes. In the AdS C-metric one takes this boundary to be at $x + y = \varepsilon_c$, where $\varepsilon_c \ll 1$. The background reference spacetime is described by (37), subjected to $-1 \leq x \leq 1$, $y \geq -x$ and (38). In this background spacetime we take the boundary Σ_t^∞ to be at $x + y = \varepsilon_0$, where $\varepsilon_0 \ll 1$.

We are now interested in writing the intrinsic metric in the boundary Σ_t^∞ (the 2-surface $t = \text{const}$ and $x + y = \varepsilon$). In the AdS C-metric (1) one performs the coordinate transformation

$$\phi = \frac{2}{|\mathcal{G}'(x_n)|} \tilde{\phi}, \quad t = \frac{2}{\mathcal{F}'(y_A)} \tilde{t} \quad (44)$$

in order that $\Delta\tilde{\phi} = 2\pi$, and the analytic continuation of \tilde{t} has period 2π , i.e., $\Delta\tilde{\tau} = 2\pi$. Furthermore, one takes

$$x = x_s + \varepsilon_c \chi, \quad y = -x_s + \varepsilon_c (1 - \chi) \quad (45)$$

where $0 \leq \chi \leq 1$. By making the evaluations up to second order in ε_c (since higher order terms will not contribute to the Hamiltonian in the final limit $\varepsilon_c \rightarrow 0$), the intrinsic metric on the boundary Σ_t^∞ is then

$$ds_{(\Sigma_t^\infty)}^2 \sim \frac{2}{A^2 \varepsilon_c \mathcal{G}'(x_s)} \left[\frac{d\chi^2}{2\chi} \right]$$

$$+ \left| \frac{\mathcal{G}'(x_s)}{\mathcal{G}'(x_n)} \right|^2 \left(2\chi + \varepsilon_c \frac{\mathcal{G}''(x_s)}{\mathcal{G}'(x_s)} \chi^2 \right) d\tilde{\phi}^2 \Big]. \quad (46)$$

Analogously, for the background spacetime (37) one sets

$$x = -1 + \varepsilon_0 \chi, \quad y = 1 + \varepsilon_0 (1 - \chi) \quad (47)$$

and the intrinsic metric on the boundary Σ_t^∞ yields

$$ds_{(\Sigma_t^\infty)}^2 \sim \frac{1}{A_0^2 \varepsilon_0} \left[\frac{d\chi^2}{2\chi} + (2\chi - \varepsilon_0 \chi^2) d\phi_0^2 \right] \quad (48)$$

These two intrinsic metrics on the boundary will agree (up to second order in ε) as long as we take the period of ϕ_0 to be given by (38) and the following matching conditions are satisfied,

$$\varepsilon_0 = -\frac{\mathcal{G}''(x_s)}{\mathcal{G}'(x_s)} \varepsilon_c, \quad (49)$$

$$A_0^2 = -\frac{[\mathcal{G}'(x_s)]^2}{2\mathcal{G}''(x_s)} A^2. \quad (50)$$

Note that the Maxwell fields of the two solutions agree trivially at the near infinity boundary Σ_t^∞ .

In what concerns the lapse function of the AdS C-metric, we evaluate it with respect to the time coordinate \tilde{t} defined in (44) and, using $[A(x+y)]^{-2} \mathcal{F} dt^2 = N^2 d\tilde{t}^2$, we find

$$N \sim \sqrt{\frac{|\Lambda|}{3}} \frac{2}{A^2 \varepsilon_c \mathcal{F}'(y_A)} \left(1 + \frac{1}{2} \frac{3}{|\Lambda|} (1 - \chi) A^2 \varepsilon_c \mathcal{G}'(x_s) \right). \quad (51)$$

Analogously, an evaluation with respect to the time coordinate t defined in (37) yields

$$N_0 \sim \frac{\mathcal{G}'(x_s)}{\mathcal{F}'(y_A)} \sqrt{\frac{|\Lambda|}{3}} \frac{1}{A_0^2 \varepsilon_0} \left(1 + \frac{3}{|\Lambda|} (1 - \chi) A_0^2 \varepsilon_0 \right). \quad (52)$$

Note that these two lapse functions are also matched by the conditions (49) and (50).

The extrinsic curvature to Σ_t^∞ as embedded in Σ_t is ${}^2K_{\mu\nu} = \sigma_\mu^\alpha h_\alpha^\beta \nabla_\beta n_\nu$ (where ∇_β represents the covariant derivative with respect to $g_{\mu\nu}$), and the trace of the extrinsic curvature is ${}^2K = g^{\mu\nu} {}^2K_{\mu\nu} = A\sqrt{\mathcal{F}(y)}$. The extrinsic curvature of the boundary embedded in the AdS C-metric is then

$${}^2K \sim \sqrt{\frac{|\Lambda|}{3}} \left(1 + \frac{1}{2} \frac{3}{|\Lambda|} (1 - \chi) A^2 \varepsilon_c \mathcal{G}'(x_s) \right), \quad (53)$$

while the extrinsic curvature of the boundary embedded in the background reference spacetime is

$${}^2K_0 \sim \sqrt{\frac{|\Lambda|}{3}} \left(1 + \frac{3}{|\Lambda|} (1 - \chi) A_0^2 \varepsilon_0 \right). \quad (54)$$

We are now in position to compute the contribution from the surface boundary term in (43). The evaluation in the AdS C-metric yields

$$\int_{\Sigma_t^\infty} d\chi d\tilde{\phi} \sqrt{\sigma} N {}^2K \sim \frac{8\pi}{|\mathcal{G}'(x_n)| \mathcal{F}'(y_A)} \frac{|\Lambda|}{3} \frac{1}{(A^2 \varepsilon_c)^2} \times \left(1 - \frac{1}{2} \frac{3}{|\Lambda|} A^2 \varepsilon_c \mathcal{G}'(x_s) \right), \quad (55)$$

while for the background reference spacetime we have

$$\int_{\Sigma_t^\infty} \sqrt{\sigma_0} d\chi d\phi_0 N_0 {}^2K_0 \sim \frac{8\pi [\mathcal{G}'(x_s)]^2}{|\mathcal{G}'(x_n)| \mathcal{F}'(y_A)} \frac{|\Lambda|}{3} \frac{1}{(2A_0^2 \varepsilon_0)^2} \times \left(1 - \frac{3}{|\Lambda|} A_0^2 \varepsilon_0 \right). \quad (56)$$

From the matching conditions (49) and (50) we conclude that these two boundary terms are equal. Hence, the surface term and the Hamiltonian (43) vanish. The Euclidean action (41) of the nonextreme AdS instanton that mediates pair creation of nonextreme black holes is then simply

$$I_{\text{nonext}} = -\frac{1}{4} (\Delta \mathcal{A}_{\text{ac}} + \mathcal{A}_{\text{bh}}), \quad (57)$$

while the Euclidean action (41) of the extreme AdS instanton that mediates pair creation of nonextreme black holes is just given by

$$I_{\text{ext}} = -\frac{1}{4} \Delta \mathcal{A}_{\text{ac}}. \quad (58)$$

Thus as occurs in the Ernst case, in the flat C-metric case, in the de Sitter case and in the dS C-metric case, the pair creation of nonextreme black holes is enhanced relative to the pair creation of extreme black holes by a factor of $e^{\mathcal{A}_{\text{bh}}}$.

In the next two subsections we will explicitly compute (57) using the results of Sec. II A, and (58) using the results of Sec. II B. In Sec. III C we analyze the pair creation rate of black holes discussed in Sec. II C. Remark that the only horizons that contribute with their areas to (57) and (58) are those that belong to the instanton responsible for the pair creation, i.e., only those horizons that are in the Euclidean sector of the instanton will make a contribution.

The domain of validity of our results is the particle limit, $mA \ll 1$, for which the radius of the black hole, $r_+ \sim m$, is much smaller than the typical distance between the black holes at the creation moment, $\ell \sim 1/A$ (this value follows from the Rindler motion $x^2 - t^2 = 1/A^2$ that describes the uniformly accelerated motion of the black holes).

A. Pair creation rate in the nonextreme case with $m = q$

In the nonextreme case, the instanton has two horizons in its Euclidean section, namely the acceleration horizon

at $y = y_A$ and the black hole horizon at $y = y_+$ (the horizons y'_A and y_- do not belong to the instanton, see Fig 1). The black hole horizon covers the whole range of the angular coordinate x , $x_s \leq x \leq x_n$, and its area is

$$\begin{aligned} \mathcal{A}_{\text{bh}} &= \int_{y=y_+} \sqrt{g_{xx}g_{\phi\phi}} dx d\phi \\ &= \frac{1}{A^2} \int_{\Delta\phi} d\phi \int_{x_s}^{x_n} \frac{dx}{(x+y_+)^2} \\ &= \frac{4\pi}{A^2 |\mathcal{G}'(x_n)|} \frac{x_n - x_s}{(x_n + y_+)(x_s + y_+)}, \end{aligned} \quad (59)$$

where y_+ is given in (10), x_s and x_n are defined by (16), and $\Delta\phi$ is given by (18).

The acceleration horizon, y_A defined in (10), of the nonextreme AdS instanton covers the angular range $-y_A \leq x \leq x_n$ (i.e., it is not present in the vicinity of the south pole), and is noncompact, i.e., its area is infinite. We have to deal appropriately with this infinity. In order to do so, we first introduce a boundary at $x = -y_A + \varepsilon_c$ ($\varepsilon_c \ll 1$), and compute the area inside of this boundary, which yields

$$\begin{aligned} \mathcal{A}_{\text{ac}}^c &= \int_{y=y_A} \sqrt{g_{xx}g_{\phi\phi}} dx d\phi \\ &= \frac{1}{A^2} \int_{\Delta\phi} d\phi \int_{-y_A+\varepsilon_c}^{x_n} \frac{dx}{(x+y_A)^2} \\ &= -\frac{4\pi}{A^2 |\mathcal{G}'(x_n)|} \left(\frac{1}{x_n + y_A} - \frac{1}{\varepsilon_c} \right), \end{aligned} \quad (60)$$

When we let $\varepsilon_c \rightarrow 0$, the term $1/\varepsilon_c$ diverges, and the acceleration horizon has an infinite area. This area is renormalized with respect to the area of the acceleration horizon of the background reference spacetime (37). This background reference spacetime has an acceleration horizon at $y = \sqrt{1 - |\Lambda|/(3A_0^2)}$ which is the direct counterpart of the above acceleration horizon of the nonextreme AdS instanton, and it covers the angular range $-\sqrt{1 - |\Lambda|/(3A_0^2)} \leq x \leq 1$. The area inside a boundary at $x = -\sqrt{1 - |\Lambda|/(3A_0^2)} + \varepsilon_0$ is

$$\begin{aligned} \mathcal{A}_{\text{ac}}^0 &= \int_{y=\sqrt{1-\frac{|\Lambda|}{3A_0^2}}} \sqrt{g_{xx}g_{\phi\phi}} dx d\phi_0 \\ &= \frac{1}{A_0^2} \int_{\Delta\phi_0} d\phi_0 \int_{-\sqrt{1-\frac{|\Lambda|}{3A_0^2}}+\varepsilon_0}^1 \frac{dx}{\left(x + \sqrt{1-\frac{|\Lambda|}{3A_0^2}}\right)^2} \\ &= -\frac{2\pi}{A_0^2 |\mathcal{G}'(x_n)|} \left(\frac{1}{1 + \sqrt{1-\frac{|\Lambda|}{3A_0^2}}} - \frac{1}{\varepsilon_0} \right), \end{aligned} \quad (61)$$

where in the last step we have replaced $\Delta\phi_0$ by (38). When $\varepsilon_0 \rightarrow 0$, the term $1/\varepsilon_0$ diverges, and thus the area of this background acceleration horizon is also infinite. Now, we have found that the intrinsic metrics at the near infinity boundary match together if (49) and (50) are satisfied. Our next task is to verify that these matching

conditions between (ε_c, A) and (ε_0, A_0) are such that the divergent terms in (60) and (61) cancel each other. It is straightforward to show that $\Delta\mathcal{A}_{\text{ac}} = \mathcal{A}_{\text{ac}}^c - \mathcal{A}_{\text{ac}}^0$ yields a finite value if

$$2A_0^2 \varepsilon_0 = A^2 \varepsilon_c \mathcal{G}'(x_s). \quad (62)$$

Thus, the matching conditions (49) and (50) satisfy condition (62), i.e., they indeed eliminate the divergencies in $\Delta\mathcal{A}_{\text{ac}}$. It is worthy to remark that with the choices (49) and (50) the proper lengths of the boundaries $x = -y_A + \varepsilon_c$ and $x = -1 + \varepsilon_0$, (given, respectively, by $l_c = \int \sqrt{g_{\phi\phi}} d\phi$ and $l_0 = \int \sqrt{g_{\phi_0\phi_0}} d\phi_0$) do not match. This is in contrast with the flat case [18], in which the choice of the matching parameters that avoids the infinities in $\Delta\mathcal{A}_{\text{ac}}$, also leads to $l_c = l_0$. In the AdS case, our main goal was to remove the infinities in $\Delta\mathcal{A}_{\text{ac}}$. We have achieved this aim by comparing the appropriate acceleration horizons in the instanton and in the reference background. The fact that the matching relations then lead to $l_c \neq l_0$ is not a problem at all. Replacing (49) and (50) in (61) yields for $\Delta\mathcal{A}_{\text{ac}} = \mathcal{A}_{\text{ac}}^c - \mathcal{A}_{\text{ac}}^0$ the result

$$\begin{aligned} \Delta\mathcal{A}_{\text{ac}} &= -\frac{4\pi}{A^2 |\mathcal{G}'(x_n)|} \\ &\times \left(\frac{1}{x_n + y_A} + \frac{1}{1 + \sqrt{1 - \frac{|\Lambda|}{3A^2} \frac{2|\mathcal{G}''(x_s)|}{[\mathcal{G}'(x_s)]^2}}} \frac{\mathcal{G}''(x_s)}{\mathcal{G}'(x_s)} \right), \end{aligned} \quad (63)$$

where $2|\mathcal{G}''(x_s)|/[\mathcal{G}'(x_s)]^2 < 1$.

Adding (59) and (63), and using the results of Sec. II A yields finally the total area of the nonextreme AdS instanton with $m = q$

$$\begin{aligned} \mathcal{A}_{\text{bh}}^{\text{nonext}} + \Delta\mathcal{A}_{\text{ac}}^{\text{nonext}} &= \\ &= -\frac{16\pi m^2}{\omega_+ (\omega_+^2 - 1)} \left(-\frac{\omega_+ - \omega_-}{(\omega_+ + \alpha)(\omega_- + \alpha)} + \frac{1}{\omega_+ - \alpha} \right. \\ &\quad \left. + \frac{1}{1 + \sqrt{1 - \frac{8|\Lambda|m^2}{3} \frac{3\omega_-^2 - 1}{\omega_-^2 (1 - \omega_-^2)}}} \frac{1 - 3\omega_-^2}{\omega_- (1 - \omega_-^2)} \right), \end{aligned} \quad (64)$$

where ω_+ and ω_- are defined in (16), α is given by (10), and condition (17) must be satisfied. The pair creation rate of nonextreme AdS black holes with $m = q$ is then

$$\Gamma_{\text{nonext}} \sim e^{\frac{1}{4}(\mathcal{A}_{\text{bh}}^{\text{nonext}} + \Delta\mathcal{A}_{\text{ac}}^{\text{nonext}})}. \quad (65)$$

Fixing A and Λ one concludes that the pair creation rate decreases as the mass of the black holes increases. Moreover, fixing m and Λ , in the domain of validity of our results, $mA \ll 1$ and $A > \sqrt{|\Lambda|/3}$, as A increases the pair creation rate increases. Hence, the general behavior of the pair creation rate of nonextreme black holes with $m = q$ in the AdS case is analogous to the corresponding behavior in the flat case [18] (see also Appendix A).

B. Pair creation rate in the extreme case ($y_+ = y_-$)

In the extreme AdS case, the instanton has a single horizon, the acceleration horizon at $y = y_A$, in its

Euclidean section, since $y = y_+$ is an internal infinity. The pair creation rate of extreme AdS black holes with $y_+ = y_-$ is then

$$\Gamma_{\text{ext}} \sim e^{\frac{1}{4}\Delta\mathcal{A}_{\text{ac}}^{\text{ext}}}, \quad (66)$$

where $\Delta\mathcal{A}_{\text{ac}}^{\text{ext}}$ is given by (63), with y_A defined in (23), x_s and x_n given by (29), and condition (27) must be satisfied. The pair creation rate decreases as the mass of the black holes increases, and the pair creation rate increases when A increases. The general behavior of the pair creation rate of extreme black holes as a function of m and A in the AdS case is also analogous to the behavior of the flat case, discussed in [18] (see also Appendix A).

C. Pair creation rate in the nonextreme case with $m \neq q$

The nonextreme instantons, that mediate the pair creation of nonextreme black holes with $m \neq q$ (including the case $q = 0$), have two horizons in their Euclidean section, namely the acceleration horizon at $y = y_A$ and the black hole horizon at $y = y_+$. The pair creation rate of nonextreme AdS black holes with $m \neq q$ is then $\Gamma \sim e^{(\mathcal{A}_{\text{bh}}^{\text{nonext}} + \Delta\mathcal{A}_{\text{ac}}^{\text{nonext}})/4}$, with $\mathcal{A}_{\text{bh}}^{\text{nonext}}$ given by (59) and $\Delta\mathcal{A}_{\text{ac}}^{\text{nonext}}$ given by (63), subjected to the results found in Sec. II C. The pair creation rate decreases as the mass of the black holes increases, and the pair creation rate increases when A increases.

IV. SUMMARY AND DISCUSSION

We have studied in detail the quantum process in which a cosmic string breaks in an anti-de Sitter (AdS) background and a pair of black holes is created at the ends of the string. The energy to materialize and accelerate the black holes comes from the strings tension. The analysis of this process in a flat background ($\Lambda = 0$) has been carried in [18], while in a de Sitter background ($\Lambda > 0$) it has been done in [28]. In an AdS background this is the only study done in the process of production of a pair of correlated black holes with spherical topology. We remark that in principle our explicit values for the pair creation rates also apply to the process of pair creation in an external electromagnetic field, with the acceleration being provided in this case by the Lorentz force instead of being furnished by the string tension. Indeed, there is no AdS Ernst solution, and thus we cannot discuss analytically the process. However, physically we could in principle consider an external electromagnetic field that supplies the same energy and acceleration as our strings and, from the results of the $\Lambda = 0$ case (where the pair creation rates in the string and electromagnetic cases agree), we expect that the pair creation rates found in this paper do not depend on whether the energy is being provided by an external electromagnetic field or by a string.

It is well known that the AdS background is attractive, i.e., an analysis of the geodesic equations indicates that particles in this background are subjected to a potential well that attracts them (see also Appendix B). Therefore, if we have a virtual pair of black holes and we want to turn them real, we will have to furnish a sufficient force that overcomes this cosmological background attraction. We then expect that pair creation is possible only if the strings tension and the associated acceleration A is higher than a critical value. We have confirmed that this is indeed the case: in the AdS background, black holes can be pair produced only with an acceleration higher than $\sqrt{|\Lambda|/3}$.

We have constructed the saddle point solutions that mediate the pair creation process through the analytic continuation of the AdS C-metric, and we have explicitly computed the pair creation rate of the process. The AdS pair creation rate reduces to the corresponding one of the flat case [18] when we set $\Lambda = 0$. We have concluded that, for a pair of black holes that is subjected to a fixed Λ and A backgrounds, the pair creation probability decreases when the mass or charge of the black holes increases. Moreover, when we fix the mass and the charge of the black holes, the probability they have to be pair created increases when the acceleration provided by the string increases. These results are physically expected, as an heuristic derivation done in Appendix B confirms. This process has also a clear analogy with a thermodynamical system, with the mass density of the string (that is proportional to A) being the analogue of the temperature T . Indeed, from the Boltzmann factor, $e^{-E_0/(k_B T)}$ (where k_B is the Boltzmann constant), one knows that a higher background temperature makes the nucleation of a particle with energy E_0 more probable. Equivalently, a higher acceleration provided by the string makes the creation of the black hole pair more probable.

For the benefit of comparison, in Fig. 3 we schematically represent the general behavior of the black hole pair creation rate Γ as a function of the acceleration A provided by the strings, when a cosmic string breaks in the three cosmological constant backgrounds. In a flat background [see Fig. 3.(a)], the pair creation rate is zero when $A = 0$ [18]. In this case, the flat C-metric reduces to a single black hole, and since we are studying the probability of pair creation, the corresponding rate is naturally zero. This does not mean that a single black hole cannot be materialized from the quantum vacuum, it only means that this latter process is not described by the C-metric. The creation probability of a single black hole in a hot bath has been considered in [44]. In a dS background [see Fig. 3.(b)], the pair creation rate is not zero when $A = 0$ [28]. This means that even in the absence of the string, the positive cosmological constant is enough to provide the energy to materialize the black hole pair [12]. If in addition one has an extra energy provided by the string, the process becomes more favorable [28]. In the AdS case [see Fig. 3.(c)], the negative cosmological constant makes a negative contribution to the process, and black

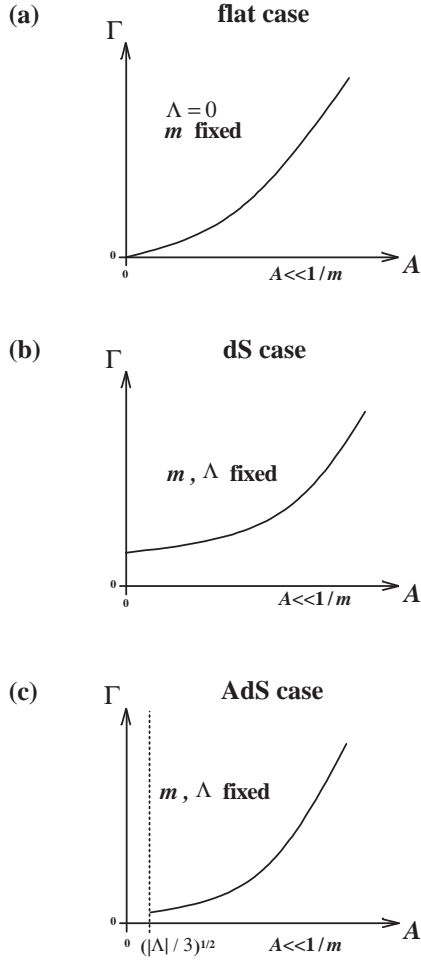


FIG. 3: General behavior of the black hole pair creation rate Γ as a function of the acceleration A provided by the strings, when a cosmic string breaks: (a) in a flat background ($\Lambda = 0$) [18], (b) in a dS background ($\Lambda > 0$) [28], and (c) in an AdS background ($\Lambda < 0$).

hole pair creation is possible only when the acceleration provided by the strings overcomes the AdS background attraction. The branch $0 < A \leq \sqrt{|\Lambda|/3}$ represents the creation probability of a single black hole when the acceleration provided by the broken string is not enough to overcome the AdS attraction, and was not studied in this paper.

We have also verified that (as occurs with pair creation in other backgrounds) the pair production of nonextreme black holes is enhanced relative to the pair creation of extreme black holes by a factor of $e^{S_{\text{bh}}}$, where $S_{\text{bh}} = \mathcal{A}_{\text{bh}}/4$ is the gravitational entropy of the black hole.

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APPENDIX A: PAIR CREATION RATE IN THE FLAT CASE

In a flat background ($\Lambda = 0$), the analysis of the process of pair creation of black holes when a cosmic string breaks has been analyzed by Hawking and Ross [18]. In this case, there is a direct $\Lambda = 0$ counterpart of the nonextreme AdS instanton and of the extreme AdS instanton discussed in our AdS case. These instantons describe pair creation of nonextreme (with $m = q$) and extreme black holes with $y_+ = y_-$, respectively [18]. The total area of these $\Lambda = 0$ instantons can be found in [18], and can be obtained by taking the direct $\Lambda = 0$ limit of (59) and (63), together with the replacement $y_A \mapsto -x_s$. This procedure yields that (59) also holds in the $\Lambda = 0$ case, while $\Delta\mathcal{A}_{\text{ac}}$ becomes

$$\Delta\mathcal{A}_{\text{ac}} = -\frac{4\pi}{A^2|\mathcal{G}'(x_n)|} \left(\frac{1}{x_n - x_s} + \frac{1}{2} \frac{\mathcal{G}''(x_s)}{\mathcal{G}'(x_s)} \right). \quad (\text{A1})$$

In [18] the explicit numerical value of \mathcal{A}_{bh} and $\Delta\mathcal{A}_{\text{ac}}$ has not been computed. We will do it here.

For the $\Lambda = 0$ nonextreme case, the discussion of Sec. II A applies generically, as long as we set $\Lambda = 0$ in the corresponding equations. With this data we find the explicit value of the total area of the nonextreme flat instanton

$$\mathcal{A}_{\text{bh}} + \Delta\mathcal{A}_{\text{ac}} = -\frac{\pi}{A^2} \frac{-1 + 4mA + \sqrt{1 - (4mA)^2}}{\sqrt{1 - (4mA)^2}}. \quad (\text{A2})$$

In the particle limit, $mA \ll 1$, the above relation reduces to $-\pi m/(4A)$ and the mass density of the string is given by $\mu \sim mA$. The pair creation rate is then $\Gamma \sim e^{-\pi m^2/\mu}$. Thus, as occurs with the AdS case, the pair creation rate decreases when m increases and the rate increases when A or μ increase [see Fig. 3.(a)].

We remark that for $mA \sim 1$, and as occurs in the corresponding AdS case, the pair creation rate associated to (A2) starts decreasing when A increases. This is a physically unexpected result since a higher acceleration provided by the string background should favor the nucleation of a fixed black hole mass. The sector $mA \sim 1$ must then be discarded and the reason is perfectly identified: the domain of validity of the rates is $mA \ll 1$, for which the radius of the black hole, $r_+ \sim m$, is much smaller than the typical distance between the black holes at the creation moment, $\ell \sim 1/A$ (this value follows from the Rindler motion $x^2 - t^2 = 1/A^2$ that describes the uniformly accelerated motion of the black holes). So, for $mA \sim 1$ one has $r_+ \sim \ell$ and the black holes start interacting with each other.

In what concerns the pair creation rate of extreme and nonextreme with $m = q$ $\Lambda = 0$ black holes, an explicit computation shows that its general behavior with A and m is also similar to the one of the AdS case, i.e., the rate decreases when m or q increase, and the rate increases when A increases.

APPENDIX B: HEURISTIC DERIVATION OF THE PAIR CREATION RATES

In order to clarify the physical interpretation of the results, in this Appendix we heuristically derive some results discussed in the main body of the paper. In particular, we find heuristically the pair creation rates.

In an AdS background, pair creation of black holes is possible only when the acceleration provided by the strings satisfies $A > \sqrt{|\Lambda|/3}$. To understand this result one can argue as follows. In general, the time-time component of the gravitational field is given by $g_{00} = 1 - 2\Phi$, where Φ is the Newtonian potential. In the AdS space-time, one has $\Phi = -|\Lambda|r^2/6$ and its derivative yields the force per unit mass or acceleration of the AdS spacetime, $A_{\text{AdS}} = -|\Lambda|r/3 \sim -\sqrt{|\Lambda|/3}$, where we have replaced r by the characteristic AdS radius $(|\Lambda|/3)^{-1/2}$. The minus sign indicates that the AdS background is attractive and thus, if one wants to have a pair of accelerated black holes driving away from each other, the cosmic string will have to provide a sufficient acceleration A that overcomes the AdS background attraction, i.e., $A > |A_{\text{AdS}}|$.

An estimate for the black hole pair creation probability can be given by the Boltzmann factor, $\Gamma \sim e^{-E_0/W_{\text{ext}}}$, where E_0 is the energy of the system that nucleates and

$W_{\text{ext}} = F\ell$ is the work done by the external force F , that provides the energy for the nucleation, through the typical distance ℓ separating the created pair. First we ask what is the probability that a black hole pair is created in a $\Lambda = 0$ background when a string breaks. This process has been discussed in [18] (see also Appendix A) where it was found that the pair creation rate is $\Gamma \sim e^{-m/A}$. In this case, $E_0 \sim 2m$, where m is the rest energy of the black hole, and $W_{\text{ext}} \sim A$ is the work provided by the strings. To derive $W_{\text{ext}} \sim A$ one can argue as follows. The acceleration provided by the string is A , the characteristic distance that separates the pair at the creation moment is $1/A$ (see Appendix A), and the characteristic mass of the system is A by the Compton relation. Thus, the characteristic work is $W_{\text{ext}} = \text{mass} \times \text{acceleration} \times \text{distance} \sim AAA^{-1}$. So, from the Boltzmann factor we indeed expect that the creation rate of a black hole pair when a string breaks in a $\Lambda = 0$ background is given by $\Gamma \sim e^{-m/A}$.

Now we ask what is the probability that a string breaks in an AdS background and a pair of black holes is produced. As we saw just above, the presence of the AdS background leads in practice to a problem in which we have a net acceleration that satisfies $A' \equiv \sqrt{A^2 - |\Lambda|/3}$, this is, Λ makes a negative contribution to the process. Heuristically, we may then apply the same arguments that have been used in the last paragraph, with the replacement $A \rightarrow A'$. At the end, the Boltzmann factor tells us that the creation rate for the process is $\Gamma \sim e^{-m/\sqrt{A^2 - |\Lambda|/3}}$. So, given m and Λ , when the acceleration provided by the string grows the pair creation rate increases, as the explicit calculations done in the main body of the paper show.

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